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# Application Note: Using Lenterra Shear Stress Sensors to Measure Viscosity

Guidelines for using Lenterra's shear stress sensors for in-line, real-time measurement of viscosity in pipes, thin channels, and high-shear mixers.

## Shear Stress and Viscosity

Shear stress is a force that acts on an object that is directed parallel to its surface. Lenterra's RealShear™ sensors directly measure the wall shear stress caused by flowing or mixing fluids. As an example, when fluids pass between a rotor and a stator in a high-shear mixer, shear stress is experienced by the fluid and the surfaces that it is in contact with. A RealShear™ sensor can be mounted on the stator to measure this shear stress, as a means to monitor mixing processes or facilitate scale-up.

Shear stress and viscosity are interrelated through the shear rate (velocity gradient) of a fluid:

$$\tau = \mu \frac{\partial u}{\partial y} = \mu \dot{\gamma} .$$

Here $\tau$ is the shear stress, $\dot{\gamma}$ is the shear rate, $\mu$ is the dynamic viscosity, $u$ is the velocity component of the fluid tangential to the wall, and $y$ is the distance from the wall.

When the viscosity of a fluid is not a function of shear rate or shear stress, that fluid is described as "Newtonian." In non-Newtonian fluids the viscosity of a fluid depends on the shear rate or stress (or in some cases the duration of stress). Certain non-Newtonian fluids behave as Newtonian fluids at high shear rates and can be described by the equation above. For others, the viscosity can be expressed with certain models.

Lenterra's RealShear™ sensor directly measures shear stress at a wall ($\tau_w$ at $y = 0$). To determine viscosity the value of the shear rate at the wall must be independently known. The viscosity is then given by:

$$\mu = \frac{\tau_w}{\dot{\gamma}_w}$$

If the shear rate is known or can be modeled, a RealShear™ sensor can be used as a real-time, in-line viscometer. Measuring viscosity in this way uses the same principle as "cup-and-bob" viscometers in which a cylinder is rotated inside a cup, submerged in the fluid under test. This powerful capability removes the need for further costly instrumentation when viscosity needs to be monitored in-line.

## Viscosity Measurement in High-Shear Mixers

High-shear mixers are used across numerous industries, including for the production of pharmaceuticals, food, and cosmetics.[1] They can provide significantly shorter mixing cycles to radically improve throughput compared with conventional mixers. HSMs are often of the rotor-stator type, in which one element (the rotor) rotates in close proximity (as small as 0.2 mm) to a stationary element (the stator). Mix components that pass between them experience high shear stress.

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[1] S. Shelley, "High-shear mixers: still widely misunderstood," *Chemical Engineering Progress*, 11, 7-12, 2007.

For high-shear mixers, the known rotor geometry and rate of rotation can be used to calculate the velocity gradient. Several assumptions can be made:

1. A “no slip” condition exists (the fluid velocity at each surface is equal to the velocity of that surface).
2. The velocity gradient profile is linear, i.e. the shear rate ( $\dot{\gamma} = \partial u / \partial y$) is the same for all distances from the wall $y$. This is typically a good assumption for HSMs due to the very small gaps between rotors and stators.

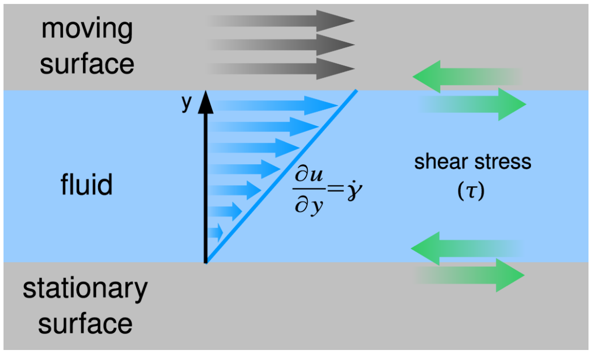

Illustration of shear stress and shear rate (velocity gradient) in a fluid between two surfaces moving relative to one another (as in rotor-stator mixers). Shear stress is exerted on the fluid by the surfaces and vice versa.

With these two assumptions, the shear rate at the stator is simply the tangential velocity of the rotor (the “tip speed”) divided by the gap distance between it and the stator.

$$\dot{\gamma}_w = \frac{u_{rotor}}{y_{gap}}$$

The rotor velocity can be calculated from its diameter, $d$, and the rate of rotation (RPM).

$$u_{rotor} = \pi \frac{f_{RPM}}{60} d$$

Using the direct wall shear stress measurement from a RealShear™ sensor, $\tau_w$, the viscosity can be calculated as:

$$\mu = \frac{60}{\pi} \frac{y_{gap}}{f_{RPM} d} \tau_w$$

The software supplied with Lenterra’s LOC-series controllers can be configured with user input parameters to automatically calculate and display the viscosity in real time.

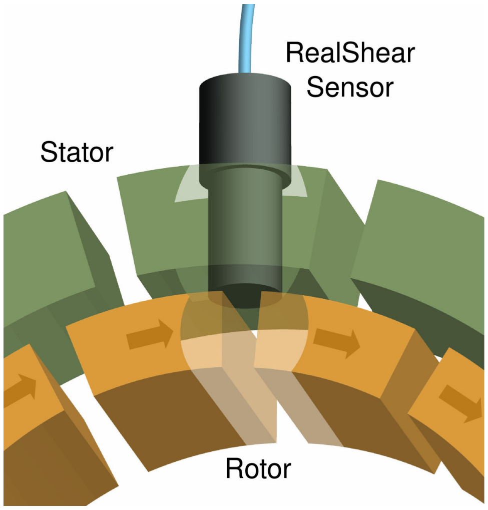

Illustration of sensor positioning for shear stress measurement.

**To calculate the viscosity in a high-shear mixer from wall shear stress measurements, the following information should be known:**

1. **Diameter of the rotor**
2. **Distance between rotor tip and the inner wall of the stator**
3. **Tip speed (can be calculated from the rate of rotation)**

## Viscosity Measurement in Pipes and Other Channels

Measurement of the wall shear stress can also be used to determine the viscosity of fluids flowing through pipes or other channels. Just as in the case of high-shear mixers, in order to calculate the viscosity from the measured wall shear stress, the wall shear rate must be known. Formulas exist that implicitly incorporate the wall shear rate for various channel geometries and can be used to calculate the viscosity based on the measured wall shear stress. These formulas are valid under the following assumptions:

1. The flow is laminar.
2. The fluid is Newtonian.
3. The flow is fully developed (the velocity gradient no longer changes as the flow continues). This condition can be assumed to occur at a distance into a pipe equal to several pipe diameters, or a distance into a thin channel equal to several channel heights.

Other approaches are available to calculate the viscosity when the flow is turbulent and when the fluid is non-Newtonian.

*Circular Cross-Section Pipes*

Viscosity ($\mu$) can be calculated from the wall shear stress ($\tau_w$) for flow in a pipe with a circular cross-section with the following formula:

$$\mu = \frac{\tau_w r}{4U}.$$

Here $U$ is the average flow velocity, and $r$ is the inner radius of the pipe. To calculate the average flow velocity from the flow rate, simply divide the volumetric flow rate ($Q$) by the cross-sectional area of the pipe ($A$):

$$U = Q/A.$$

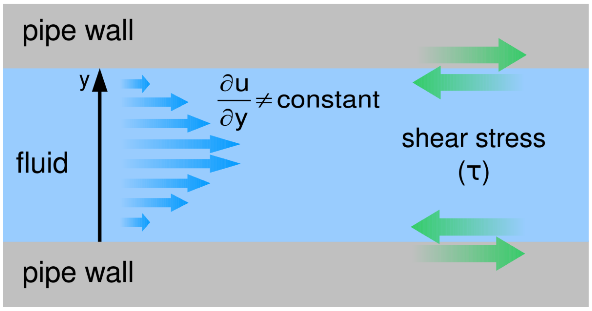

Illustration of shear stress and shear rate (velocity gradient) in a fluid flowing in a pipe. The fluid velocity is highest in the center of the pipe, and zero at the pipe wall. Shear stress is exerted on the fluid by the pipe wall and vice versa.

*Thin Rectangular Cross-Section Channels*

Viscosity can also be calculated in thin channels (in which the height of the channel is much less than the width), such as those found at the exit zone of extruders. For a rectangular thin channel the viscosity is

$$\mu = \frac{\tau_w h}{6U}$$

where $U$ is the average fluid velocity, and $h$ is the height of the channel.

**To calculate the viscosity in a pipe or thin channel from wall shear stress measurements, the following information should be known:**

1. **Dimensions of the channel (pipe inner diameter or height of thin channel)**
2. **Average flow velocity (can be calculated from the flow rate)**

## Temperature Measurement

For both Newtonian and non-Newtonian fluids, viscosity is dependent on temperature. Depending on the particular fluid, even small changes of temperature can result in significant variation of viscosity. As a consequence, accurate interpretation of viscosity measurements should be accompanied by temperature measurements.